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Atomic Clock

The history of advancements in time measurement is closely tied to the history of scientific discoveries. As early as the 17th century, Ole Rømer, by comparing the results of a pendulum clock with those of an astronomical clock, (i) deduced the finite speed of light, (ii) discovered thermal expansion, and (iii) introduced a new temperature scale, known as the Rømer scale. Around this time, the pendulum clock was also used to detect deviations in the Earth's surface from a perfect sphere. Today, atomic clocks, alongside gravitational wave detectors, are among the most precise measuring instruments.

In this project:

a) Learn how we maintain the global time on Earth.

b) Learn how the Ramsey interferometry works and how precise it is

c) Can the Schrödinger cat be used to improve the determination of tim

Definition of second and its realisations

You have to learn the basics about time measurements (use any sources of knowledge, including the official ones, like `https://www.bipm.org/en/time-metrology`):

1. What is the current SI definition of the second?
2. Read definitions of accuracy, stability of atomic clocks and the Allan deviation. Find any scientific papers about the performance of the atomic clock from around 2000 and from the last five years, both having budgets of uncertainties. Compare them.
3. What devices are used to measure time? What role is played by the local oscillator and what local oscillators are used nowadays?
4. Learn the basic inforamtion) about the Ramsey interferometry and what is the Mach–Zehnder interferometer.

To understand how the atomic clock works you will start with a single atom description, and generalize it to N atoms.

Single atoms part:

1. Remind yourself of the basics of the quantum description of two-level systems: the Pauli matrices $\hat{\sigma}_x$, $\hat{\sigma}_y$, $\hat{\sigma}_z$, the Bloch sphere, the Stokes vector.
2. Read about the Ramsey, and the Mach–Zehnder interferometry.

   Let's assume that an atom is initiated in the state $|\psi(0)\rangle := |0\rangle$ (the south pole of the Bloch sphere), and then it is a subject of the following transformations:

   - the so-called $\pi/2$ pulse, given by the unitary operation:

$$U_{\pi/2} = e^{-i\pi\hat{\sigma}_x/4} \tag{1}$$

- Free evolution:
  $$U(t) = e^{-i\hat{H}t/\hbar}, \tag{2}$$
  where $\hat{H} = E_0|0\rangle\langle 0| + E_1|1\rangle\langle 1|$,
- Next $\pi/2$ pulse.
- Measurement of $\hat{\sigma}_z$.

Plot the average value $\langle\hat{\sigma}_z(t)\rangle$ as a function of time, in the state at the end of the procedure, i.e.

$$|\psi(t)\rangle := U_{\pi/2}\, U(t)\, U_{\pi/2}|\psi(0)\rangle. \tag{3}$$

Let's assume that the atom was the Cesium atom, and the states $|0\rangle$ and $|1\rangle$ are the ones from the SI definition of the second.

Imagine that the experimentalist estimates the value of σ_z at the end of the protocol to $s_z = 0.5 \pm 0.1$.

- What time passed until the measurement? Assume that $\pi/2$ pulses are instantaneous.
- At what time t did the measurements happen, given that (for instance according to previous estimations) already 1000 full oscillation passed? What is the uncertainty of the estimated time, at which the measurement took place?
- The wavefunction $|\psi(t)\rangle$ has the meaning of the probability distribution. Mimic the experiment in the following way: use the probability distribution $P(0) := |\langle 0|\psi(t)\rangle|^2$ to draw the internal state of the atom. Repeat it 20 times when you expect that $\langle\sigma_z\rangle$ is (i) 0, (ii) 1/2, (iii) 1. To estimate time, one would take the arithmetic average of measurements – this is the estimation s_z of $\langle\sigma_z\rangle$ and then inverse the analytical formula $\langle\sigma_z\rangle(t)$ to estimate time. Perform such analysis for drawn (="measured") values. What will be the estimation of t according to measured values? How do they differ from the real time?

  How many measurements is used to estimate s_z?

  At what point one should measure σ_z to obtain the most precise estimation of time?
- The simplest experimental formula to the uncertainty of the measured quantity is $\mathcal{O}$ is
  $$\Delta t = \Delta\mathcal{O}/|\partial_t \left\langle \hat{\mathcal{O}}(t) \right\rangle| \tag{4}$$
  What is the optimal point of the measurement of time according to this formula, for $\mathcal{O} = \hat{\sigma}_z$? If this is unclear consult first [2].

3. Repeat the calculations and the final plot but for N atoms.

   Assume that each atom is a subject to the $\pi/2$ pulses, and the Hamiltonian is: $\hat{H}_N = E_0 \sum_{j=1}^N |0\rangle_j\langle 0|_j + \sum_{j=1}^N E_1|1\rangle_j\langle 1|_j$

   The measured quantity is:
   $$\hat{S}_z = \frac{1}{2}\sum_{j=1}^N \sigma_{z,j} = \frac{1}{2}\sum_{j=1}^N \left(|1\rangle_j\langle 1|_j - |0\rangle_j\langle 0|_j\right), \tag{5}$$
   which is just the number of atoms in $|1\rangle$ minus the number of atoms in $|0\rangle$ over 2.

   Repeat the whole analysis from the previous part (the one about a single atom), but for this new scenario.

Entangled states

One of the idea to improve the precision is to change protocol. Repeat all previous state assuming, that

- Initial state is so called GHZ state (example of the Schrödinger cat state) with N atoms:

$$|\psi_{\mathrm{GHZ}}(0)\rangle = \frac{1}{\sqrt{2}} \left(|000..0\rangle - |1111..11\rangle\right) \tag{6}$$

- The free evolution is given by the same Hamiltonian as before, i.e. the Hamiltonian for free Cesium atoms, not being subjected to any force or potential.
- One is measuring parity operators:

$$\hat{\Pi}_x = \prod_{j=1}^{N} \sigma_x = (-1)^{\hat{S}_z} \tag{7}$$

Current studies

1. Read about the squeezed states in [3] and the related experiments of the Nobel prize winner D. Wineland.
2. Read about the measurements of gravitation [1]
3. What is the optical atomic clock?
4. Can one use atomic clock to measure the gravitational waves? Scan literature about this subject

Presentation

Please include in your presentation:

- Definitions of accuracy, stability of atomic clocks and the Allan deviation. How accurate and stable are current devices?
- How global time UTC is "updated"? Visit and explore the webpages of offices responsible for global time synchronization
- Progress about using clocks in the study of gravity. Can one use clocks to measure gravitational waves?
- Do quantum entanglement helps? What are the prospects?

References

[1] G. W. Biedermann, X. Wu, L. Deslauriers, S. Roy, C. Mahadeswaraswamy, and M. A. Kasevich. Testing gravity with cold-atom interferometers. *Phys. Rev. A*, 91:033629, Mar 2015.

[2] G. Toth and I. Apellaniz. Quantum metrology from a quantum information science perspective. *arXiv*, May 2014.

[3] D. J. Wineland, J. J. Bollinger, W. M. Itano, and D. J. Heinzen. Squeezed atomic states and projection noise in spectroscopy. *Phys. Rev. A*, 50:67–88, Jul 1994.