- Declaration: Oct 4, 2024
- Project supervisor: Emilia Witkowska, `ewitk@ifpan.edu.pl`
- Consultation: Oct 15, 2024 (Tuesday) at 13:15, hall D
  If needed – another meeting in the week Oct, 14-18
- Presentation: Jan. 24, 2025

The mini-project aims to become familiar with quantum computers and computations. It starts with searching the literature on example platforms for quantum computing implementation at work. Find as many as possible, prepare a list of them and discuss the one made of atoms in more detail.

Getting to Know Some Quantum Gates

One of the most common ways of thinking about quantum computation is in terms of quantum circuits: An overall quantum computation is decomposed into smaller building blocks, typically referred to as gates. This task aims to familiarize oneself with some important gates and decompose an overall unitary operation into gates.

(a) Recall what is the classical NOT gate. Write down the (unitary) matrix for the single-qubit gate that implements an analogue of the classical NOT. That is, the gate should act as $|0\rangle \to |1\rangle$, $|1\rangle \to |0\rangle$.

(b) Write down the (unitary) matrix for the two-qubit controlled-NOT (CNOT) gate, which implements a computational basis flip on the second qubit controlled on the first qubit being active. That is, the gate should act as $|00\rangle \to |00\rangle$, $|10\rangle \to |11\rangle$, $|11\rangle \to |10\rangle$.

(c) Recall the Hadamard gate H. Describe a quantum circuit with two gates that, starting at $|00\rangle$, prepares the maximally entangled state $(|00\rangle + |11\rangle)/\sqrt{2}$. Prove that your circuit acts as desired.

(d) Recall what are the SWAP gate and Bell states for a pair of qubits. Calculate an explicit matrix form for the swap gate. What does this gate do to a pair of qubits in a Bell state? Why is this answer not surprising?

Classical Circuit Complexity

Classical computers perform any operation by combining a few elementary building blocks (what we call gates) into more complex operations. As soon as we fix a specific set of available gates (classical computers can do this using only the NAND operation between neighbouring bits, for example), we say we have fixed a circuit model. (Sometimes, one additionally assumes restrictions on the circuit layout, but we ignore this here.) Given this model, we can define the circuit complexity of a function. This is the minimum number of gates from the given available set that is sufficient to construct the desired function. In other words: A function is said to have circuit complexity at most G (w.r.t. some circuit model) if there is a circuit (in that model) with at most G gates that implements the given function.

In this task, you will investigate how rare the property of having "small" circuit complexity is.

(a) Recall the classical Boolean functions. Consider a classical circuit model in which circuits are composed of arbitrary two-bit gates whose input bits are either a constant (0 or 1), an entry of an input string or its negation, or an output of some other gate. Let $G \in \mathbb{N}$. Show that there are at most $16G(G + 2n + 1)2G$ classical Boolean functions mapping n bits to 1 bit that have circuit complexity at most G in this model.

Bases for Unitary Quantum Gates

Usually, when talking about quantum computing, people refer to the manipulation of pure quantum states of a certain number of qubits to perform a task of interest. In this setting, the gates (operations that transform the state) are described by unitary matrices. When writing matrices down, the first natural choice is to do it in the computational basis (the input-output relationship between states $|i\rangle$ and $|j\rangle$ under a given operation), but one could choose a different basis. We want to explore this a little in the following two exercises and look at different bases for the space of operators.

(a) Recall the Bell state for two qubits. Find explicit expressions for the four computational basis states of a two-qubit system in terms of superpositions of the four Bell states.

(b) How can the four Bell states be converted into four distinguishable states on a computational basis?